Conformal nature of the Hawking radiation.

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Abstract

String theory usually represents quantum black holes as systems whose statistical mechanics reproduces Hawking's thermodynamics in a very satisfactory way. Complicated brane theoretical models are worked out, as quantum versions of Supergravity solutions.

These models are then assumed to be in thermal equilibrium: this is a little cheating, because one is looking for an explanation of the seeming thermodynamical nature of black holes, so these cannot be *assumed* to be finite temperature systems!

In the model presented here, the black body spectrum arises with no statistical hypothesis as an approximation of the unitary evolution of microscopic black holes, which are always described by a $1+1$ conformal field theory, characterized by some Virasoro $\mathfrak{Vir}$ or $\mathfrak{Vir} \oplus \overline{\mathfrak{Vir}}$ algebra.

At the end, one can state that *the Hawking-thermodynamics of the system is a by-product of the algebraic* $\mathfrak{Vir} \oplus \overline{\mathfrak{Vir}}$*-symmetric nature of the event horizon.* This is *the central result* of the present work.

I. INTRODUCTION.

When a quantum radiation in a well known initial state $|in\rangle_{\rm rad}$ is sent into a black hole, then a black body radiation is received back, so that some *thermalization machine* seems to be at work, turning the quantum pure state $|in\rangle_{\rm rad}$ into the quantum mixture $\rho^{out}_{\rm rad}(T_{\rm BH})$, a canonical ensemble at the Hawking temperature $T_{\rm BH}$. If things work this way, the presence of black holes in our universe immediately forces us to change drastically the basic principles of Quantum mechanics!

More, a black hole can be attributed a finite amount of entropy, the Beckenstein-Hawking entropy, which is shown to be an essentially geometrical object (in classical terms), and behaves exactly as thermodynamical entropies, tending to grow indefinitely within classical processes (when the positive energy hypothesis is done), and decreasing when the hole evaporates, while the total entropy of the universe grows on due to the thermal radiation from the dying hole.

String theorists have constructed representations of the quantum states of black holes whose evolution is assumed to be unitary. The quality check of such stringy models consists in verifying that their quantum *statistical* mechanics fits perfectly with the values of the thermodynamical parameters evaluated for the classical solution corresponding to them. The agreement between these models and supergravity classical solutions is perfect (see [1], [2], [3], [4] and many others).

This wonderful agreement, anyway, does not explain fully the mechanism of the Hawking effect: stringy models for black holes have the right temperature when they are supposed to be thermodynamical objects to be treated statistically... But, why have they to be treated statistically, even if they are describing microscopic black holes?

I think that, just as black holish thermodynamical laws and quantities emerge outside any explicitly statistical context in the classical general relativity, it must be possible to produce quantum models which show thermodynamical properties without any need of postulated thermal equilibrium: this behavior must be shared by any believable quantum model of

microscopic black holes, because it characterizes the very presence of an event horizon.

The question addressed in this work is *whether it is possible to obtain a thermal radiation from string-modeled black holes without supposing explicitly their quantum state to be a finite temperature distribution*. The answer seems to be that *it is possible*, at least for the black holes presented here.

The paper is organized as follows.

In Section II the "ideological" basis of our construction is presented, and the effective D-string model proposed for four and five dimensional SUGRA black holes. In Section III the mathematical result is presented, and the non-thermodynamical derivation of the Hawking effect explained. Section IV contains a formula which relates the Hawking temperature with the algebraic characteristics of the theory at hand, and suggests some connection between the present results and Carlip's approach in [5] and [17]. Section V is devoted to conclusions.

II. THE SUMMING-AVERAGING HIDING MECHANISM.

The construction of Quantum mechanics in a curved spacetime is deeply influenced by the presence of event horizons, because such causal structures diminish our capacity of observing the world: what we can see from outside of the degrees of freedom and observables of the black hole, is only few charges, the ADM mass and angular momenta.

This condition of concealment of the "other" quantum numbers of the black hole is very particular: they cannot be observed, but they cannot be "anything"! As we will see in the models presented, the structure of the quantum Hilbert space $\mathbf{H}_{\mathrm{BH}}$ must be very well tuned to reproduce the correct Hawking radiation. All this amount of microscopic information sits "behind the horizon", but influences deeply the emitted radiation which, even if thermal, still has a precise memory of what is inside (in fact the T_{BH} here predicted is a function of the main quantities defining the quantum theory).

A. The horizon-centric ideology.

Suppose to deal with a process during which the microscopic black hole emits quanta. At the beginning of this process, the black hole quantum state is some $|\psi_{\mathrm{BH}}(i)\rangle$. Then the system emits a quantum with momentum k, due to some interaction $V_{\mathrm{int}}(k)$, and undergoes a transition to another quantum state $|\psi_{\mathrm{BH}}(f)\rangle$. The transition rate of the process is evaluated in terms of the matrix element

$$\mathsf{S}_{if} = \langle \psi_{\mathrm{BH}}(f)| V_{\mathrm{int}}(k) |\psi_{\mathrm{BH}}(i)\rangle . \tag{2.1}$$

Now, the approach presented here is to consider the initial and final quantum states of the black hole as hidden by the horizon, so the process must be calculated as inclusive with respect to all the possible final states with ADM quantities $(M', J', Q', ...)$, and averaged over all the possible initial states with ADM quantities $(M, J, Q, ...)$; this rate reads:

$$\Gamma^{\mathrm{hidden}}_{f,i} = \sum_{f,\langle i\rangle} |\mathsf{S}_{if}|^2 \equiv \frac{1}{\mathbf{N}(M)} \sum_{i,f} |\mathsf{S}_{if}|^2 \tag{2.2}$$

(here $\mathbf{N}(M) = \dim\ker\left(\hat{M} - M\mathbf{1}\right)$).

The idea is than:

- *the right quantum system modelling a given microscopic black hole must be constructed in such a way that the quantity $\Gamma^{\mathrm{hidden}}_{f,i}$ in (2.2) is a thermal distribution with the correct values of the parameters of the corresponding classical solution.*

First of all, in order for $\Gamma^{\mathrm{hidden}}_{f,i}$ to be a thermal distribution, the degeneracies must be wide enough so that averaging over the initial states of mass equal to M, and summing over the final states of mass M', is very similar to taking a thermodynamical limit in the number of degrees of freedom [6]. A black hole is thought of as a highly excited system (see [7], [8] and [9]), and we need models in which $\mathbf{N}(M)$ is a rapidly growing function of M. There are systems with this property, whose high level degeneracy exactly matches that of black holes: in general we will see that $1 + 1$ dimensional conformal field theories share this feature.

The criterion of thermal nature for $\Gamma_{f,i}^{\text{hidden}}$ is important because it leads to *a relationship between the emission temperature of the black hole and the central charge of the CFT of the horizon* [10]. This relationship seems to be important because it appears in every model explored here (and no presentation of this "general coincidence" was known to me before)[1].

If the condition of thermal nature on $\Gamma_{f,i}^{\text{hidden}}$ holds in the form expressed before, the correspondence point can be smoothly reinterpreted: suppose that the system, which generated a black hole collapsing, is a certain quantum system Q. Typically, this will be seen as a black hole when its ADM energy M is so big that:

$$r_s\left(M\right) > \ell_{\mathsf{Q}}, \tag{2.3}$$

being ℓ_{Q} the physical size of Q (see e.g. [7]). Roughly speaking, when $r_s\left(M\right) > \ell_{\mathsf{Q}}$ *there exists an event horizon* which (at least classically) hides the system, allowing for the observation of ADM quantities only.

Now, the mass starts to decrease by Hawking radiation: suppose that this process can go on till the Schwarzschild radius becomes smaller than the physical size of Q, as

$$\ell_{\mathsf{Q}} > r_s\left(M\right). \tag{2.4}$$

From this point, there is *no event horizon*, and the external observer can describe the system with the same precision that would be possible in absence of any gravitational complication at all.

The picture is the following: when condition (2.3) is met, many particulars of the system are hidden by the horizon, and in this situation any quantum process must be studied as in

[1]It is very important to say that the present results about $\Gamma_{f,i}^{\text{hidden}}$ are very near to those obtained treating the systems in a *microcanonical statistical ensemble*: there appears a first degree of approximation in which the ensemble can be considered as canonical [11], while the microcanonical nature of the statement $|\psi_{\text{BH}}\rangle \in \ker\left(\hat{M} - M\mathbf{1}\right)$ is revealed when a finer approximation is considered; at this further level, the behavior of $\Gamma_{f,i}^{\text{hidden}}$ resembles that of the spectrum found in [12] for the explicitly microcanonical approach.

(2.2), because ADM quantum numbers (M, J, Q) are everything that an external observer can hope to measure.

On the contrary, when condition (2.4) holds, there is no event horizon preventing the observer to measure completely the ket $|\psi_{\mathrm{BH}}\rangle$: external observers *can choose* to perform an inclusive measure, but this is simply up to them.

The correspondence principle suggests that

- *the "optional" inclusive calculation performed in the case (2.4), and its "obligatory" version that must be worked out in the case (2.3), have to agree at the correspondence point*

$$\ell_{\mathsf{Q}} = r_s\,(M)\,. \tag{2.5}$$

This implies that the nature of the spectrum of the (uncollapsed, horizonless) quantum system must be such to produce a thermal spectrum when it undergoes inclusive processes, in the suitable approximations (then, the condition on $\Gamma^{\mathrm{hidden}}_{f,i}$ as expressed before); not only: the quantities characterizing this thermodynamics, that is temperature and entropy, must agree with the correspondent quantities for the collapsed system in the *decollapsing limit*

$$r_s\,(M) \to (\ell_{\mathsf{Q}})_+\,.$$

This conception describes the evaporation process very smoothly; anyway, it is naive in a sense, because of *the key role played everywhere by the horizon*: maybe, the fault of this model is its belief that a classical horizon goes on existing at quantum level.

In a completely classical context, horizons are spacetime curtains that prevent external observers from receiving any information at all from inside the black holes; in the semiclassical context of QFT's in curved backgrounds, horizons can stop only ordered information, because Hawking thermal radiation can reach the future infinity.

It is expectable that in a fully quantum theory of spacetime, even ordered information can tunnel across the horizons and reach the future infinity (this is what results as those in

[12] are telling us): if that tunnelling process were possible for ordered information, horizons would essentially disappear in exact quantum gravity (as substantial walls can disappear to tunnelling particles in Quantum mechanics).

B. Horizon QFT for string theoretical black holes.

A black hole is a strongly coupled system, and one could expect it is impossible to describe it in a perturbative way; nevertheless, string theory is able to give, at least for nearly extremal black holes, a perturbative quantum mechanical description by turning them into *dual* weakly coupled bound systems of D-branes. The curved near horizon SUGRA spacetime is thus described by a system of quantum objects living in a flat background, emitting and absorbing (massless) strings to which they are weakly coupled[2].

In [1] the five- and the four-dimensional SUGRA black holes are described as dual to bound systems of D-branes, that are respectively D1- and D5-branes in the first case, D2-, D6-branes and solitonic 5-branes in the second; four or five spacelike directions are compactified, but one of the compactification radii is much bigger than the others:

$$R \gg R_i. \tag{2.6}$$

This fact allows for a description of such systems as *D-strings*.

In the low energy approximation, the starting point to study D-strings is the *Born-Infeld action* [13]

$$S_{BI} = -T_{\mathrm{F}} \int\limits_{\mathbb{V}_2^{\mathrm{D}}} d^2\xi e^{-\phi(X)} \sqrt{-\det\left[G_{\mathrm{mn}}\left(X\right) + B_{\mathrm{mn}}\left(X\right) + 2\pi\alpha' F_{\mathrm{mn}}\right]}, \tag{2.7}$$

that reduces simply to the Nambu-Goto-like functional

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[2]This way of working, which was found to be right in a range of cases wider than what was originally expected, has been clarified in terms of Maldacena conjecture and AdS/CFT correspondence.

$$S_{BI} = -T_{\mathrm{F}} \int\limits_{\mathbb{V}_2^{\mathrm{D}}} d^2\xi e^{-\phi_0} \sqrt{-\det\left[G_{\mathrm{mn}}(X)\right]} \tag{2.8}$$

when $B_{\mathrm{mn}} = 0$ and $F_{\mathrm{mn}} = 0$.

Let us study the case of emission of a *gravitational wave*, with string frame metric

$$G_{\mu\nu} = e^{\frac{1}{2}\phi_0} g_{\mu\nu} = e^{\frac{1}{2}\phi_0}\left(\eta_{\mu\nu} + 2\kappa h_{\mu\nu}\right), \tag{2.9}$$

where the components of h are very small, and T_{mn} is the pull-back of a tensor $T_{\mu\nu}$ along the worldsheet of the D-string as $T_{\mathrm{mn}} = T_{\mu\nu}\partial_{\mathrm{m}}X^{\mu}\partial_{\mathrm{n}}X^{\nu}$.

The D-string action is then expanded in the gravitational field as follows:

$$S_{BI} = -\frac{T_{\mathrm{F}}}{\sqrt{g_{\mathrm{closed}}}} \int\limits_{\mathbb{V}_2^{\mathrm{D}}} d^2\xi \sqrt{-\det \eta_{\mathrm{mn}}(X)} +$$

$$+\frac{T_{\mathrm{F}}}{\sqrt{g_{\mathrm{closed}}}} \int\limits_{\mathbb{V}_2^{\mathrm{D}}} d^2\xi \frac{\kappa}{\sqrt{-\det \eta_{\mathrm{mn}}(X)}} \left[\eta_{\sigma\sigma}(X) h_{\tau\tau}(X) - 2\eta_{\tau\sigma}(X) h_{\tau\sigma}(X) + \eta_{\tau\tau}(X) h_{\sigma\sigma}(X)\right] + ...$$

Let us now suppose X to be a small fluctuation around some *classical solution* X_0 *of the equations of motion* $\ddot{X}_0 - X_0'' = 0$: this X_0 is chosen so that

$$X_0^0 = \tau,\ X_0^1 = \sigma,\ \vec{X}_0 = \vec{X}_0(\sigma, \tau),\ \sqrt{-\det \eta_{\mathrm{mn}}(X_0)} = 1. \tag{2.10}$$

The "exact" field X differs from X_0 only for *small fluctuations*[3] $X - X_0 = \mathbb{O}\left(h^{\frac{1}{2}}\right)$, the consistent approximation reads:

$$S_{BI} = -\frac{T_{\mathrm{F}}}{\sqrt{g_{\mathrm{closed}}}} \int\limits_{\mathbb{V}_2^{\mathrm{D}}} d^2\xi \sqrt{-\det \eta_{\mathrm{mn}}(X)} +$$

$$+\frac{T_{\mathrm{F}}\kappa}{\sqrt{g_{\mathrm{closed}}}} \int\limits_{\mathbb{V}_2^{\mathrm{D}}} d^2\xi \left[\eta_{\sigma\sigma}(X_0) h_{\tau\tau}(X) - 2\eta_{\tau\sigma}(X_0) h_{\tau\sigma}(X) + \eta_{\tau\tau}(X_0) h_{\sigma\sigma}(X)\right] + ... \tag{2.11}$$

[3]In this position the power p of $X - X_0 = \mathbb{O}(h^p)$ is chosen in the spirit of neglecting $o(h)$ everywhere.

This action is formally the same as that of a free string perturbed by some interaction piece

$$S_{\text{int}}[X] = -\frac{T_{\text{F}}\kappa}{\sqrt{g_{\text{closed}}}} \int\limits_{\mathbb{V}_2^{\text{D}}} d^2\xi h_{\mu\nu}(X)\left(\dot{X}^{\mu}\dot{X}^{\nu} - X'^{\mu}X'^{\nu}\right), \tag{2.12}$$

which does coincide formally with the *graviton emission vertex for the elementary closed string theory* [14]. The Nambu-Goto string is quantized *à la Polyakov*, by replacing its radical action with a quadratic one that is completely identical to the F-string action:

$$S_{\text{free}}[X] = \frac{T_{\text{D}}}{2} \int\limits_{\mathbb{V}_2^{\text{D}}} d^2\xi \left(\dot{X}^{\mu}\dot{X}_{\mu} - X'^{\mu}X'_{\mu}\right), \; \frac{T_{\text{F}}}{\sqrt{g_{\text{closed}}}} = T_{\text{D}}. \tag{2.13}$$

The D spacetime components of the field X^{μ} are redundant, and this is fixed by assuming

$$X^0 = \tau, \; X^1 = \sigma$$

(static gauge), leaving $D-2$ independent variables only.

III. QUASI THERMAL EMISSIONS.

The idea that the Hawking radiation can be obtained as a purely microscopic, non thermodynamical, effect by using $1+1$ CFT's occurred to us[4] studying a very interesting phenomenon (discovered by D. Amati and J.G. Russo in 1999, see [11]) in which *a unitarily evolving quantum system emits thermal radiation*, when inclusive rates are evaluated. This system is *the fundamental bosonic string*: such rates will be referred to as *quasi-thermal elementary string emission* (QUATESE for short).

A. Quasi-thermal emission from elementary strings.

The action of an elementary bosonic string is:

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[4]This work is partially the synthesis of my PhD thesis [10], whose advisor was Professor Giorgio Immirzi, of the State University of Perugia.

$$S=\frac{T_{\mathrm{F}}}{2}\int\limits_{\mathbb{V}_2} d^2\sigma\eta^{\mathrm{ab}}\partial_{\mathrm{a}}X^{\mu}\partial_{\mathrm{b}}X_{\mu}. \tag{3.1}$$

In open bosonic string theory the mass is quantized and each single state is characterized by the *partition* $\{N_n\}$:

$$M^2=\frac{1}{\alpha'}\left(N-1\right),\ \sum_n nN_n=N. \tag{3.2}$$

It is seen that:

$$N\gg 1\ \Rightarrow\ \dim\ker\left(\hat{N}-N\mathbf{1}\right)=p_N^{(d)}\simeq\frac{K}{N^{\frac{1}{4}(d+3)}}e^{\pi\sqrt{\frac{2Nd}{3}}}. \tag{3.3}$$

If the spacetime dimension is D, the number d of *free independent string fields* is $d=D-2$.

Let an open string decay from the level N emitting a photon of momentum k and energy ω, down to the level $N'\neq N$. The transition takes place due to the insertion of a photon vertex $V\left(k,\xi\right)=\xi_{\mu}\dot{X}^{\mu}\left(0\right)e^{ik\cdot X(0)}$, being ξ the photon polarization. The decay rate is:

$$d\Gamma_{i,k,f}=\left|\langle f|\,V\left(k,\xi\right)|i\rangle\right|^2 V\left(S^{D-2}\right)\omega^{D-3}d\omega,\ V\left(S^{D-2}\right)=\frac{2\pi^{\frac{D-1}{2}}}{\Gamma\left(\frac{D-1}{2}\right)}. \tag{3.4}$$

If the only observed outputs are the photon $|\xi,k\rangle$ and the masses of the final and of the initial states, the whole decay probability is the sum of the $\left|\langle f|\,V\left(k,\xi\right)|i\rangle\right|^2$'s over all the final states, and the average over the initial states of the assigned levels. This is an *inclusive decay rate*:

$$d\Gamma_i\left(\omega\right)=\frac{1}{p_N^{(d)}}\sum_{i,f}\left|\langle f|\,V\left(k,\xi\right)|i\rangle\right|^2 V\left(S^{D-2}\right)\omega^{D-3}d\omega. \tag{3.5}$$

The evaluation of this $d\Gamma_i\left(\omega\right)$ is performed under the conditions

$$N,N'\gg 1,\ N-N'\ll N,N',\ N-N'\ll\sqrt{N},\sqrt{N'}, \tag{3.6}$$

that will be referred to as *canonical limit*; the very interesting result is that $d\Gamma_i\left(\omega\right)$ has a black body form:

$$d\Gamma_i\left(\omega\right)=2\left(\xi^*\cdot\xi\right)\alpha' MV\left(S^{D-2}\right)\omega^{D-2}\frac{\exp\left(-\frac{\omega}{T_H}\right)}{1-\exp\left(-\frac{\omega}{T_H}\right)}d\omega,\ T_H=\frac{1}{2\pi\sqrt{\alpha'}}\sqrt{\frac{6}{D-2}}. \tag{3.7}$$

Amati and Russo's work explains that the emission rate $d\Gamma_i(\omega)$ becomes a thermal rate as soon as the summing-averaging operation $\sum_{f,\langle i\rangle}$ is performed, in the determinant condition (3.6): a more detailed study of the physical sense of (3.6) is going to be presented as soon as possible in [10].

The origin of this thermal nature is the wide degeneracy of the string levels, expressed in (3.3), which turns out to be an effect of the $1+1$ conformal symmetry dominating the string physics. That the quasi-thermal[5] behavior of the elementary string emission is a by-product of the Virasoro symmetry is written explicitly in the form of the parameter T_H (check equation (3.7)): $D-2$ is *the number of free independent fields*, and it can be directly equated to *the central charge of the algebra* $\mathfrak{Vir}$ *in the CFT with fixed lightcone gauge* [14]

$$T_H = \frac{2}{\ell}\sqrt{\frac{6}{\mathsf{C}_{\text{free}}}},\ \ell = 4\pi\sqrt{\alpha'}. \tag{3.8}$$

The rate for the emission of a soft massless particle with momentum k from a *closed string*, decaying from an initial state in $\ker\left(\hat{N}_R - N_R\mathbf{1}\right)\otimes\ker\left(\hat{N}_L - N_L\mathbf{1}\right)$ to a final one in $\ker\left(\hat{N}_R - N_R'\mathbf{1}\right)\otimes\ker\left(\hat{N}_L - N_L'\mathbf{1}\right)$, is calculated immediately from what has been done for the open string, due to the worldsheet-chiral factorization.

The mass shell relationship reads:

$$\alpha' M^2 = 4\left(N_R - 1\right) + \alpha' Q_+^2 = 4\left(N_L - 1\right) + \alpha' Q_-^2,\ N_R - N_L = m_0 w_0, \tag{3.9}$$

where m_0 and w_0 are the Kaluza-Klein and winding numbers of the string along the compactified dimension of radius R, and:

$$Q_\pm = \frac{m_0}{R} \pm \frac{w_0 R}{\alpha'}. \tag{3.10}$$

Due to the chiral factorization of states and vertices the scattering matrix reads:

$$\mathsf{S}_{if} = \langle R_f|\, V_R\left(\tfrac{k}{2},\xi^R\right)|R_i\rangle\,\langle L_f|\, V_L\left(\tfrac{k}{2},\xi^L\right)|L_i\rangle = \mathsf{S}_{if}^R\mathsf{S}_{if}^L, \tag{3.11}$$

[5] I say "quasi-thermal" because $d\Gamma_i(\omega)$ is a Planckian distribution only in the canonical limit (3.6).

so that the summed-averaged emission rate is

$$\sum_{f,\langle i\rangle} d\Gamma_{i,k,f} = \sum_{f',\langle i'\rangle} \left|\mathsf{S}^{R}_{i'f'}\right|^2 \sum_{f'',\langle i''\rangle} \left|\mathsf{S}^{L}_{i''f''}\right|^2 V\left(S^{D-2}\right)\omega^{D-3}d\omega. \tag{3.12}$$

Step by step, what has been done for the open string can be repeated for the two chiral halves, getting:

$$\sum_{f,\langle i\rangle} \left|\mathsf{S}^{R,L}_{if}\right|^2 \simeq K\frac{\omega}{2}\left(\xi^*_{R,L}\cdot\xi_{R,L}\right)M\alpha'\frac{\exp\left(-\frac{\omega}{2T_{R,L}}\right)}{1-\exp\left(-\frac{\omega}{2T_{R,L}}\right)}. \tag{3.13}$$

These are *black body spectra*: the left- and the right-moving temperatures are:

$$T_L = \frac{\sqrt{M^2-Q_-^2}}{a\sqrt{\alpha'}M},\ T_R = \frac{\sqrt{M^2-Q_+^2}}{a\sqrt{\alpha'}M} \tag{3.14}$$

and can be expressed in terms of the central charges as:

$$T_L = \frac{1}{\pi\alpha' M}\sqrt{\frac{6\left(N_L-1\right)}{\mathsf{C}_{\text{free}}}},\ T_R = \frac{1}{\pi\alpha' M}\sqrt{\frac{6\left(N_R-1\right)}{\tilde{\mathsf{C}}_{\text{free}}}}. \tag{3.15}$$

By approximating M^{-1} as the M_0^{-1} of the unexcited string at rest, with $M_0 = L_{\text{F}}T_{\text{F}}$ and $T_{\text{F}} = \frac{1}{2\pi\alpha'}$, one can write:

$$T_L = \frac{2}{L_{\text{F}}}\sqrt{\frac{6\left(N_L-1\right)}{\mathsf{C}_{\text{free}}}},\ T_R = \frac{2}{L_{\text{F}}}\sqrt{\frac{6\left(N_R-1\right)}{\tilde{\mathsf{C}}_{\text{free}}}}. \tag{3.16}$$

The closed string decay rate reads:

$$\begin{gathered} d\Gamma = \frac{K^2\left(\xi^*_R\cdot\xi_R\right)\left(\xi^*_L\cdot\xi_L\right)M^2\alpha'^2}{4} \\ \frac{\exp\left(-\frac{\omega}{2T_R}\right)}{\left[1-\exp\left(-\frac{\omega}{2T_R}\right)\right]}\frac{\exp\left(-\frac{\omega}{2T_L}\right)}{\left[1-\exp\left(-\frac{\omega}{2T_L}\right)\right]}V\left(S^{D-2}\right)\omega^{D-1}d\omega. \end{gathered} \tag{3.17}$$

This decay rate can be rewritten as the product of a black body spectrum with temperature:

$$T^{-1} = \frac{1}{2}\left(T_R^{-1}+T_L^{-1}\right), \tag{3.18}$$

times the suitable *grey body factor*:

$$\sigma\left(\omega,T\right) = \frac{1-\exp\left(-\frac{\omega}{T}\right)}{\left[1-\exp\left(-\frac{\omega}{2T_R}\right)\right]\left[1-\exp\left(-\frac{\omega}{2T_L}\right)\right]}\omega. \tag{3.19}$$

B. QUATESE for D-brane modeled black holes.

The low energy physics of a D-string is governed by actions which are formally equal to those of the F-string. The naive conjecture is that everything could work the same way for the D- as for the F-string, with the replacement:

$$T_{\mathrm{F}} \rightarrow T_{\mathrm{D}}. \tag{3.20}$$

If this system decays from a very excited level to some slightly less energetic level, the emission will be quasi thermal again, when summed over the final polarizations and averaged over the initial ones.

Unfortunately things are not so simple: the elementary string mass levels in (3.9) are not those of a D-string. The spectrum of non BPS states of a D-superstring reads [2]:

$$\begin{cases} M_{\mathrm{D}} \simeq LT_{\mathrm{D}} + \dfrac{4\pi}{L}\left(N_R - \delta_R\right) - \dfrac{2\pi}{L} m_0 w_0, \\ \\ M_{\mathrm{D}} \simeq LT_{\mathrm{D}} + \dfrac{4\pi}{L}\left(N_L - \delta_L\right) + \dfrac{2\pi}{L} m_0 w_0, \end{cases} \tag{3.21}$$

(being L the length of the D-string[6] and $\delta_{R,L}$ corrective constant zero point terms due to the supersymmetry), while the elementary string spectrum gives mass levels of the form:

$$\begin{cases} M_{\mathrm{F}} = \sqrt{8\pi T_{\mathrm{F}}\left(N_R - \delta_R\right) + \dfrac{4\pi^2 w_0^2 m_0^2}{L^2} + 4\pi T_{\mathrm{F}} m_0 w_0 + L^2 T_{\mathrm{F}}^2}, \\ \\ M_{\mathrm{F}} = \sqrt{8\pi T_{\mathrm{F}}\left(N_L - \delta_L\right) + \dfrac{4\pi^2 w_0^2 m_0^2}{L^2} - 4\pi T_{\mathrm{F}} m_0 w_0 + L^2 T_{\mathrm{F}}^2}. \end{cases} \tag{3.22}$$

In the *long string hypothesis* $L\sqrt{T_{\mathrm{F}}} \gg 1$ the mass in (3.22) can be expanded as:

$$\begin{cases} M_{\mathrm{F}} = T_{\mathrm{F}} L + \dfrac{2\pi}{L} m_0 w_0 + \dfrac{4\pi}{L}\left(N_R - \delta_R\right) + \mathbb{O}\left(\dfrac{1}{T_{\mathrm{F}} L^3}\right), \\ \\ M_{\mathrm{F}} = T_{\mathrm{F}} L - \dfrac{2\pi}{L} m_0 w_0 + \dfrac{4\pi}{L}\left(N_L - \delta_L\right) + \mathbb{O}\left(\dfrac{1}{T_{\mathrm{F}} L^3}\right), \end{cases} \tag{3.23}$$

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[6] The approximated equality symbol $\simeq$ is due to the fact that this would be exact only if the massless open strings travelling along the D-string did not interact.

so in this regime there exists a perfect duality between the elementary string mass and the D-string one (this is the S-duality presented in [15]).

More rigorously, let us consider what changes in Amati and Russo's argument if the F-string is replaced by such a D-string. The quantum numbers $N_{R,L}$ are:

$$\begin{cases} N_R = \dfrac{L}{4\pi}M_{\mathrm{D}} - \dfrac{L^2}{4\pi}T_{\mathrm{D}} - \dfrac{w_0 m_0}{2} + \delta_R, \\ \\ N_L = \dfrac{L}{4\pi}M_{\mathrm{D}} - \dfrac{L^2}{4\pi}T_{\mathrm{D}} + \dfrac{w_0 m_0}{2} + \delta_L. \end{cases} \tag{3.24}$$

The expressions of $N'_{R,L}$ in terms of M'_{D} are formally equal (w_0 and m_0 do not change). With some kinematics, M'_{D} is related to M_{D} (the D-string being initially at rest), and for small ω one has:

$$N'_R = N_R - \frac{L}{4\pi}\omega,\ N'_L = N_L - \frac{L}{4\pi}\omega. \tag{3.25}$$

The inclusive process transition rate reads:

$$\sum_{f,\langle i\rangle} d\Gamma_{i,\omega,f} = \sum_{f',\langle i'\rangle} \left|\mathsf{S}^R_{i'f'}\right|^2 \sum_{f'',\langle i''\rangle} \left|\mathsf{S}^L_{i''f''}\right|^2 V\left(S^{D-2}\right)\omega^{D-3}d\omega,$$

due to the chiral factorization: the rest is pure mathematics, the net result is again:

$$\sum_{f',\langle i'\rangle} \left|\mathsf{S}^{R,L}_{i'f'}\right|^2 = K\frac{L\omega}{4\pi}\left(\xi^*_{R,L}\cdot\xi_{R,L}\right)\frac{\exp\left(-\frac{\omega}{2T_{R,L}}\right)}{1-\exp\left(-\frac{\omega}{2T_{R,L}}\right)}, \tag{3.26}$$

a quasi-thermal emission with the temperatures:

$$T^{(\mathrm{D})}_R = \frac{2}{L}\sqrt{\frac{6N_R}{D-2}},\ T^{(\mathrm{D})}_L = \frac{2}{L}\sqrt{\frac{6N_L}{D-2}}. \tag{3.27}$$

By involving the central charges $\mathsf{C}^{(\mathrm{D})}_{\mathrm{free}} = \tilde{\mathsf{C}}^{(\mathrm{D})}_{\mathrm{free}} = D-2$, this relationship reads:

$$T^{(\mathrm{D})}_R = \frac{2}{L}\sqrt{\frac{6N_R}{\mathsf{C}^{(\mathrm{D})}_{\mathrm{free}}}},\ T^{(\mathrm{D})}_L = \frac{2}{L}\sqrt{\frac{6N_L}{\tilde{\mathsf{C}}^{(\mathrm{D})}_{\mathrm{free}}}}. \tag{3.28}$$

The grey body spectrum can be re-constructed as

$$\frac{d\Gamma}{V\left(S^{D-2}\right)d\omega} = K^2\left(\xi^*_R\cdot\xi_R\right)\left(\xi^*_L\cdot\xi_L\right)\frac{L^2}{16\pi^2}\omega^{D-2}\sigma\left(\omega,T_{\mathrm{BH}}\right)\frac{\exp\left(-\frac{\omega}{T_{\mathrm{BH}}}\right)}{1-\exp\left(-\frac{\omega}{T_{\mathrm{BH}}}\right)} \tag{3.29}$$

with

$$\sigma\left(\omega,T\right)=\frac{\left[1-\exp\left(-\frac{\omega}{T_{\mathrm{BH}}}\right)\right]\omega}{\left[1-\exp\left(-\frac{\omega}{2T_R}\right)\right]\left[1-\exp\left(-\frac{\omega}{2T_L}\right)\right]},\ \frac{1}{T_{\mathrm{BH}}}=\frac{1}{2}\left(\frac{1}{T_R}+\frac{1}{T_L}\right) \tag{3.30}$$

(this relationship agrees with (4.34) of [16]).

The result found for the D-string can now be easily generalized to the four and five dimensional black holes studied in [1] and [16]: the change is simply in the central charge of the SCFT. In the case of the five dimensional black hole the central charges are $\mathsf{C}_{\mathrm{free}}^{\mathrm{D}(1+5)}=\tilde{\mathsf{C}}_{\mathrm{free}}^{\mathrm{D}(1+5)}=6Q_1Q_5$, while for the four dimensional black hole, one has: $\mathsf{C}_{\mathrm{free}}^{\mathrm{D}(2+6)-5\mathrm{s}}=\tilde{\mathsf{C}}_{\mathrm{free}}^{\mathrm{D}(2+6)-5\mathrm{s}}=6Q_2Q_5Q_6$ [18].

IV. THE HAWKING TEMPERATURE LAW.

The emergence of a Hawking effect is explained in terms of the huge degeneracy of the quantum levels for the $\mathfrak{Vir}$- or $\mathfrak{Vir}\oplus\overline{\mathfrak{Vir}}$-symmetric systems, and it takes place when the conditions (3.6) are met. So there must be the way to predict the value of this temperature simply by assigning the conformal symmetry basics of the system at hand: here I suggest a law expressing the Hawking temperature as a function of the central charges only.

In the case of a $\mathfrak{Vir}$-symmetric theory, if $\mathsf{C}_{\mathrm{free}}$ is the value of the central charge, there is a QUATESE temperature, reading:

$$T=K\sqrt{\frac{6}{\mathsf{C}_{\mathrm{free}}}}; \tag{4.1}$$

here K is a *constant.*

In the case of the $\mathfrak{Vir}\oplus\overline{\mathfrak{Vir}}$-symmetric systems there exist two QUATESE temperatures, on left and one right-moving, which are still expressed as:

$$T_R=K\left(N\right)\sqrt{\frac{6}{\mathsf{C}_{\mathrm{free}}}},\ T_L=K\left(N\right)\sqrt{\frac{6}{\tilde{\mathsf{C}}_{\mathrm{free}}}}. \tag{4.2}$$

The spectrum is a grey body one; here K is a *function of the mass quantum level*, around which the emitting transition takes place.

In what follows the expressions corresponding to this equation (4.1) and (4.2) are singled out for systems showing quasi-thermal emission: the F- and D-strings and several kinds of black hole, and the functions $K(N)$ are put in evidence.

A. Strings and string-theoretical black holes.

The first system is the *F-string*. In the open case, we can single out the function K as:

$$T_{\text{open}}^{(\text{F})} = \frac{1}{2\pi \ell_s}\sqrt{\frac{6}{\mathsf{C}_{\text{free}}}} \Rightarrow K_{\text{open}}^{(\text{F})} = \frac{1}{2\pi \ell_s}, \ \frac{\partial}{\partial N} K_{\text{open}}^{(\text{F})} = 0. \tag{4.3}$$

In the case of the *closed* F-string, there is *a right and a left moving temperatures*: the functions K read:

$$K_R^{(\text{F})} = \frac{\sqrt{N_{R,L}-1}}{\pi \ell_s^2 M}, \ K_L^{(\text{F})} = \frac{\sqrt{N_{R,L}-1}}{\pi \ell_s^2 M}. \tag{4.4}$$

In the *long string approximation* $T^{(\text{F})}L \gg 1$ these read:

$$K_{R,L}^{(\text{F})}(N) = \frac{1}{\pi R}\sqrt{N_{R,L}-1}. \tag{4.5}$$

When the inclusive calculation is performed for the *D-string*, the functions $K(N)$ are equal as in the case of the F-string (4.5):

$$K_{R,L}^{(D)}(N) = \frac{1}{\pi R}\sqrt{N_{R,L}-1}. \tag{4.6}$$

The *five dimensional near extreme black* hole is represented by a D$(1+5)$-brane bound system [1]: when it is treated with *canonical ensemble calculations*, the values of right and left temperatures read

$$T_R^{\text{D}(1+5)}\,(\text{c.e.c.}) = \frac{1}{\pi R}\sqrt{\frac{6N_R}{\mathsf{C}^{\text{D}(1+5)}}}, \ T_L^{\text{D}(1+5)}\,(\text{c.e.c}) = \frac{1}{\pi R}\sqrt{\frac{6N_L}{\tilde{\mathsf{C}}^{\text{D}(1+5)}}}. \tag{4.7}$$

By evaluating the *inclusive emission rate* of the D$(1+5)$-system, a quasi thermal spectrum is found, with the QUATESE temperatures

$$T_R^{\text{D}(1+5)}\,(\text{i.e.r.}) = \frac{1}{\pi R}\sqrt{\frac{6N_R}{\mathsf{C}^{\text{D}(1+5)}}}, \ T_L^{\text{D}(1+5)}\,(\text{i.e.r.}) = \frac{1}{\pi R}\sqrt{\frac{6N_L}{\tilde{\mathsf{C}}^{\text{D}(1+5)}}}. \tag{4.8}$$

If one deals with the D$(2+6)$-5s-system describing the SUGRA four dimensional black hole in [1], the QUATESE temperatures would follow again the law (4.1) in the form

$$T_R^{\mathrm{D}(2+6)-5s}\,(\text{i.e.r.}) = \frac{1}{\pi R}\sqrt{\frac{6N_R}{\mathsf{C}^{\mathrm{D}(2+6)-5s}}},\; T_L^{\mathrm{D}(2+6)-5s}\,(\text{i.e.r.}) = \frac{1}{\pi R}\sqrt{\frac{6N_L}{\tilde{\mathsf{C}}^{\mathrm{D}(2+6)-5s}}}, \tag{4.9}$$

being $\mathsf{C}^{\mathrm{D}(2+6)-5s} = \tilde{\mathsf{C}}^{\mathrm{D}(2+6)-5s} = 6Q_2Q_5Q_6$, and the canonical ensemble $T_{R,L}^{\mathrm{D}(2+6)-5s}$ (c.e.) would have the same form.

B. General relativity black holes.

Someone has found a $\mathfrak{Vir}$-symmetry which characterizes the physics of a classical Einstein black hole in any dimension, arbitrarily far from extremality: this has been done by Carlip (see [5] and [17]), who showed that *the boundary physics of a stationary spacetime with an horizon is invariant under a class of diffeomorphisms of the Cauchy hypersurfaces, forming a Virasoro algebra with central charge*

$$\mathsf{C}_{\mathrm{BH}} = \frac{3\mathcal{A}}{2\pi G}, \tag{4.10}$$

being $\mathcal{A}$ the measure of the bifurcation sphere at the horizon.

Does the relationship (4.1) apply to those black holes considered by Carlip, when the role of the central charge is played by $\frac{3\mathcal{A}}{2\pi G}$, as equation (4.10) predicts?

Let us consider the *Schwarzschild classical black hole.* The horizon area reads

$$\mathcal{A} = 16\pi G^2 M^2 \;\Rightarrow\; \mathsf{C}_{\mathrm{BH}} = 24 G M^2, \tag{4.11}$$

while the Hawking temperature is

$$T_{\mathrm{BH}}^{\mathrm{Schw}} = \frac{1}{8\pi G M}\;: \tag{4.12}$$

then one has the following relationship

$$T_{\mathrm{BH}}^{\mathrm{Schw}} = \frac{1}{4\pi\sqrt{G}}\sqrt{\frac{6}{\mathsf{C}_{\mathrm{BH}}}},\; K_{\mathrm{BH}}^{\mathrm{Schw}} = \frac{1}{4\pi\sqrt{G}} \tag{4.13}$$

thus, the Hawking temperature of a Schwarzschild black hole does obey the law (4.1).

Consider the *Reissner-Nordström classical black hole*: from (4.10) one finds

$$\mathsf{C}_{\mathrm{BH}} = \frac{6r_+^2}{G} = 6G\left(2M^2 - Q^2 + 2M\sqrt{M^2 - Q^2}\right). \tag{4.14}$$

The Hawking temperature for the Reissner-Nordström solution reads:

$$T_{\mathrm{BH}}^{\mathrm{R-N}} = \frac{\sqrt{G^2M^2 - Q^2}}{2\pi\sqrt{G}\left(GM + \sqrt{G^2M^2 - Q^2}\right)}\sqrt{\frac{6}{\mathsf{C}_{\mathrm{BH}}}}. \tag{4.15}$$

To trace a relationship between the emitting strings and the charged black hole of General relativity, a comparison between slightly non BPS systems has to be made, hoping that this will quantum-protect our calculations. For *small values* of $\sqrt{G^2M^2 - Q^2}$, the factor $K_{\mathrm{R-N}}$ is expanded as:

$$K_{\mathrm{R-N}}(M) = \frac{\sqrt{G^2M^2 - Q^2}}{2\pi\sqrt{G}\left(GM + \sqrt{G^2M^2 - Q^2}\right)} = \frac{\sqrt{G^2M^2 - Q^2}}{2\pi G^{\frac{3}{2}}M} + ... \tag{4.16}$$

If some law as (3.9) is assumed

$$G^2M^2 - Q^2 = \alpha_{\mathrm{R-N}}^2 N,\ \ N \in \mathbb{N},$$

in the spirit of [9], then the relationship (4.16) becomes:

$$K_{\mathrm{R-N}}(M) = \frac{\alpha_{\mathrm{R-N}}}{2\pi G^{\frac{3}{2}}M}\sqrt{N} + ..., \tag{4.17}$$

which is quite similar to (4.6) (this $\alpha_{\mathrm{R-N}}$ is some constant introduced *ad hoc*). This relationship (4.17) suggests that *nearly extremal Reissner-Nordström black holes show a Hawking temperature that obeys the QUATESE law (4.2) if Carlip's interpretation of the area law is adopted.*

V. CONCLUSIONS.

The emission rate from F-strings becomes approximately thermal when very excited states are treated, and low energy particles emitted: if the rate is summed over the final states and averaged over the initial ones, then a Planckian distribution appears.

Can string theoretical black holes benefit of a similar mechanism? The answer is yes, because the string modeled black holes studied here can be represented as a suitable $1+1$ SCFT: this is explicitly verified for D-strings, for five dimensional holes corresponding to D$(1+5)$ systems, and for four dimensional holes corresponding to D$(2+6)$-5s systems. In all these cases, the emission temperature is a function of the central charge of the SCFT at hand, which has the same forms (4.1) and (4.2) for all these systems.

The emission rates have to be inclusive and averaged, when black holes are treated, due to the presence of the event horizon that prevents external observers from seeing the exact state of the black hole, even if its ADM charges can be measured from infinity.

The QUATESE mechanism seems to give the right answer for the Hawking temperature even outside the full superstring theory, as it is shown when the Schwarzschild and the near extremal Reissner-Nordström black holes of General relativity are examined in the spirit of [17].

The natural development of this result is to check what happens when a string falls into a black hole whose horizon physics is described by Carlip's Virasoro algebra: is the central charge of the worldsheet $\mathfrak{Vir} \oplus \overline{\mathfrak{Vir}}$ algebra of the F-string related to the increment $\delta\mathcal{A}$ of the measure of the bifurcation sphere, due to its absorption? Is the worldsheet $\mathfrak{Vir} \oplus \overline{\mathfrak{Vir}}$ algebra of every fundamental string physically "the same" algebra characterizing the black hole?

In case of a positive answer, this line of research would allow to claim for the stringy nature of elementary constituents of matter simply due to the existence of black hole, because only one dimensional objects have Virasoro algebræ among their internal symmetries, and in this vision the $1+1$ conformal symmetry would be promoted to the role of spacetime fundamental symmetry.

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